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DISCUSSION OF LONGITUDINAL MIXING MEASURED BY RADIOACTIVE TRACERS

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By Conrad P. Straub and Donald A. Pecsok; Alfred C. Ingersoll; and Harold A. Thomas, Jr., and Ralph S. Archibald

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DISCUSSION

CONRAD P. STRAUB,[7] A. M. ASCE, AND DONALD A. PECSOK[8].—The determination of flow patterns and mixing in tanks has been a problem which has confronted design engineers for many years. As stated by the authors (see "Introduction"), salt and dye methods have been used, but the results obtained by these techniques have not always been satisfactory. The use of radioactive tracers offers the engineer a convenient and simple procedure for studying the complex phenomena of hydraulic flow and evaluating some of the parameters involved. Of course, special equipment must be available for measuring the radioactivity in the pipe channel itself, or in the tanks. Equipment has been developed which makes possible the continuous monitoring of activity in pipe lines or in tank effluents. With such equipment it should be unnecessary to withdraw and measure individual samples. With future development of immersion-type counting equipment, it may be possible to measure the activity throughout the liquid in a tank, conduit, or natural body of water.

Experience gained at Oak Ridge National Laboratory (ORNL), Oak Ridge, Tenn., in the use of carrier-free and carrier-added radioiodine-131 has indicated that when carrier-free I–131 is added to activated sludges or when passed through trickling filters, much of the I–131 will be adsorbed or combined with the organic matter. These removals, which have occurred to a lesser degree with raw sewage, are at variance with the statement by the authors (see fourth paragraph preceding "Experimental Results"): "Iodide is not readily precipitated, adsorbed, or chemically changed by substances normally occurring in water and sewage." It is assumed that the authors have reference to radioactive iodine in the iodide form, in weak basic sodium sulfite solution, which is the form in which it is shipped from the Operations Division, ORNL. The ORNL studies show that the amount of radioactive I–131 removed from solution is dependent upon several factors, most pertinent of which are: The amount of organic matter in the medium used, the time of contact, pH, and the amount of carrier KI added.

Before use is made of any radioactive tracer for flow tests in tanks, natural basins, or channels in which sewage or industrial wastes may be present, it would be desirable to run jar-test studies to determine the "take-up" of the radio element by the material contained in the waste. Such studies will also provide data on the concentrations of the radioisotope that must be used. For example, if it is found that 80% to 90% of the I–131 would be removed by the organic matter present in an aeration tank under investigation, it would be necessary to increase the initial dose by a factor of 10 to obtain desirable measurable concentrations in the effluent—that is, concentrations comparable to

NOTE.—This paper by Harold A. Thomas, Jr., and Ralph S. Archibald was published in August, 1951, as *Proceedings-Separate No. 84*. The numbering of footnotes, equations, and tables is a continuation of the consecutive numbering used in the original paper.

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those that would have been obtained if there had been no removal of I–131 from solution by the organic matter. Tests with raw sewage have indicated that from 10% to 20% of the I–131 could be adsorbed on the organic matter. It is presumed that organic matter found in streams would react in a similar fashion. To reduce the uptake of activity from solution by organic matter the addition of carrier KI has been found quite effective. Dosages as large as 10 ppm of KI have been adequate for this purpose.

Since there is the possibility that I–131 may be volatilized and lost into the atmosphere, the use of this tracer is not recommended when longitudinal mixing is investigated, in tanks containing acid liquids or in acid streams.

The definition of half life given by the authors (in the third paragraph preceding "Experimental Results")—

> "The half life of an isotope is the period required for one half of the initial energy to be dissipated"—

may be confusing, although the example clarifies the statement somewhat. A preferable definition would be

> "The half life is the time required for the radioisotope to lose one-half of its total activity; that is, the time required for one half of the original atoms to disintegrate."

If the authors' definition had been stated in terms of total energy, less confusion might have resulted. Actually, the energy released per disintegration is constant and has no relation to the age of a particular radioisotope.

ALFRED C. INGERSOLL,[9] A. M. ASCE.—The authors are to be commended for opening a new channel in the field of tracer technology in sanitary engineering and for finding another peacetime use of radioactive isotopes. Radioactive "markers" have been used in many industrial applications since 1946, as in locating pipe obstructions and serving as boundary markers for successive shipments of oil in pipelines. The extension presented in this paper, to quantitative analysis of the mixing process, will provide a valuable research tool for attacking problems on the flow through mixing tanks, aeration chambers, and settling basins.

In the example of the effect of longitudinal mixing in which a chlorine contact tank is considered under the two extreme cases of no mixing and of violent mixing, it was assumed that, as a result of the violent mixing, the effluent consisted of ten portions having average detention times 0.5, 1.5, $\cdots$ 9.5 min each. Such an assumption would lead to an average detention time of only 5.0 min for the tank with a 10-min nominal detention period. Although this may not seem incompatible with reality, in view of the violent mixing which certainly causes much of the flow to be detained for less than half the nominal period, the fact is that the average detention time in the perfect mixing tank is equal to the displacement detention time, or twice that implied by the authors. The distribution curve of the individual detention times among the particles of an effluent sample can be shown to be essentially the same as the concentration-time distribution curve (Fig. 2 (*a*)), which represents the probability of

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detention time for any fluid element at the influent. The concentration-time distribution curve for the ideal mixing tank is given by the differential equation,

$$\frac{dc}{dt} = -\frac{c}{T} \quad \ldots\ldots (15)$$

which integrates into

$$\frac{c}{c_0} = e^{-t/T} \quad \ldots\ldots (16)$$

in which c is the concentration at time t (c_0 representing the initial concentration). Since the total area under this dimensionless curve is unity, the area between any two ordinates represents the proportion of the flow having detention time in this range. Next, the left side of Eq. 16 can be considered as a relative probability and the exponent (or abscissa) on the right side can be written as a function of the area, A, under the probability curve

$$\frac{t}{T} = \log_e \frac{1}{1 - A} \quad \ldots\ldots (17)$$

Eq. 3 then becomes

$$u = u_0 e^{-kTt/T} \quad \ldots\ldots (18)$$

and for the final result, with violent mixing (introducing Eq. 17 into the exponent in Eq. 18):

$$u_2 = 10^6 \int_0^1 e^{10 \log_e (1-A)} dA = \frac{10^6}{11} \quad \ldots\ldots (19)$$

which is seen to be not greatly different from the result obtained by the authors, the reason being that u_2 is determined principally by the extreme short-circuiting of the first portions of the flow to reach the effluent.

In their discussion of experimental technique (Part II) the authors state that "In general both salts and dyes may conveniently be detected in the flow if their concentration exceeds about 1/10 ppm * * *." In his experience with sodium phosphate as a tracer, in the study of model separating chambers, the writer has found concentrations of the phosphate ion of only 0.006 ppm to be clearly distinguishable with the use of a standard colorimetric technique.[10] Such concentrations were found to produce no discernible density current effects. Although sodium chloride is notorious for its density current effects, it does not seem fair to compare the Capen tests[5] with a salt tracer to the tests with radioactive tracers in Fig. 4 on the basis mainly of density currents. Of equal importance is the fact that the tests were conducted on different tanks with different inlet devices. When sodium chloride is used in the test of a settling tank the density effect often shows up as an excessive delay in the modal or mean time and the final scavenging of the salt from the tank. This occurred on tests by H. E. Babbitt, M. ASCE, and H. E. Schlenz,[11] A. M. ASCE,

[10] "Determination of Hydraulic Characteristics of Separating Chambers," by Alfred C. Ingersoll, *Proceedings*, Midwestern Conference on Fluid Dynamics, 1950, p. 393.

[5] "Study of Sewage Settling Tank Design," by C. H. Capen, Jr., *Engineering News-Record*, Vol. 99, 1927, p. 833.

[11] "Results of Tests of Sewage Treatment," by H. E. Babbitt and H. E. Schlenz, *Bulletin 198*, Univ. of Illinois Eng. Experiment Station, Part 5, 1929, pp. 54–83.

in which the peak of the distribution curve was delayed beyond the nominal detention time.

The data given in Tables 2 and 3 provide some interesting information on the flow of fluids in pipes. The percentage values for the first trace, given in Col. 5, for example, can be interpreted as giving the ratio of the average to the maximum velocity in the pipe, if it may be assumed that the first trace arrives with the velocity of the fastest part of the flow (generally considered to be along the center line of the pipe). If it be assumed, more logically, that the first trace arrives with a combined velocity, made up of the maximum instantaneous velocities of the fastest moving particles in the turbulent flow, then the values in Col. 5 would be expected to be less than the values of V/V_{max}. With the foregoing in mind, it will be illuminating to compare values from Table 2 with the best obtainable values for V/V_{max}, computed from the Nikuradse formula,

$$\frac{V}{V_{max}} = \frac{1}{1 + 1.435\sqrt{f}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

with values of the friction factor, f, given by Lewis F. Moody,[12] for commercial steel or wrought-iron pipe.

Table 4 shows that the first trace behaves in accordance with the second of the foregoing assumptions for the first of the runs considered, but there is no ready explanation for the seemingly anomalous condition in the second and third runs. It may well be that the tracer test has provided a more accurate measure of V/V_{max} than can be obtained from the Nikuradse formula. The first of the four runs given in Table 2 cannot be used in this comparison since it lies in the critical range between laminar and turbulent flow.

TABLE 4.—Comparison of First Trace Time with V/V_{max} for Pipe Flow

Reynolds number	First trace[a]	Friction factor, f	V/V_{max} (%)
9,140	73.6	0.0332	79.2
10,710	82.2	0.0324	79.5
14,210	89.4	0.0306	79.9

[a] Percentage of T from Table 2.

Given an accurate concentration-time distribution curve for the flow in a uniform reach of pipe, it should be possible to construct a reasonably valid profile of velocity distribution. This method might then be applied to such analyses in cases where it would be easier than the installation of a pitot tube.

Laminar flow in tubes affords an especially good testing ground for the radioactive tracer, for in this case the first trace should not appear in less than half the nominal time, except through molecular diffusion. The results of Table 3(*c*), at a Reynolds number of 2,180, show the percentage for the first trace to be significantly less than 50. This shows that the velocity distribution in the critical flow range is even more distorted from the uniform than for pure laminar flow. It will be valuable if the authors are able to present one run in the bona fide laminar range to prove the tracer diffusion rate. In such a run, if the first trace appears in appreciably less than 50% of the nominal time,

[12] "Friction Factors for Pipe Flow," by Lewis F. Moody, *Transactions*, A. S. M. E., November, 1944.

the radioactive tracer will be open to question as an accurate indicator of the flow.

The writer concurs with the authors in their conclusion that the modal time is not a reliable measure of discharge. It has been his experience, however, that the median time is a more reliable indicator of short-circuiting behavior than is the average time, which is always affected by the long tail of the distribution curve. This is not to contradict the authors' statement that the average time seems to be the best measure of discharge of the group.

Certainly there are many applications of tracer technology which may soon be expected to benefit from the multiple advantages of radioactive tracers. It is to be hoped that researchers in their use will be actively pursued so as to further the information now available.

HAROLD A. THOMAS, JR.,[13] AND RALPH S. ARCHIBALD,[14] J. M. ASCE.—The discussion has been gratifying in that a number of relevant points pertaining to tracer technology have been elucidated.

Mr. Ingersoll perfers a method of calculating the coliform content in a chlorine contact basin under the assumption of violent mixing, which the writers agree is rational. Indeed, it was a generalization of his line of reasoning that underlies Eqs. 5 and 7. His numerical result may be obtained by direct substitution in Eq. 7.

His thoughts on the use of the tracer test to obtain a measure of the velocity distribution are perceptive. The writers concur with Mr. Ingersoll that in the turbulent regime the maximum velocities, calculated from the time of appearance of the first trace, should be larger than those given by Eq. 20. However, tests continue to yield the seemingly anomalous results noted by Mr. Ingersoll. For example, in a carefully conducted run in a 6,400-ft reach of an old tuberculated 20-in. pipe having a friction factor, f, of 0.074, and carrying a discharge of 6.75 cu ft per sec, the first trace was observed at a time corresponding to 78.2% of the mean flow time. According to Eq. 20 this value should be no larger than 71.8%.

Part of the discrepancy, at least, may be attributed to the difficulty in timing the arrival of the first trace precisely. The first portion of the slug is highly diluted and yields counts differing only slightly from the background count. Consequently, the net count may be evaluated only with a low degree of precision. It is hoped that this question will be resolved with special counting equipment in the Sanitary Engineering Laboratory at Harvard University. Apparatus entailing a heavy iron shield for the elimination of local gamma radiation, together with a bundle of anti-coincidence tubes for screening cosmic background, yielded very low background counts (3 to 8 counts per min). Part of this remaining background was found to be due to natural radioactivity (K^{40}) in the glass of commercial GM tubes. Therefore, special tubes have been developed. Preliminary tests indicate that a considerably higher order of sensitivity and precision is attainable for tracer studies in pipes, streams, and lakes.

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Messrs. Straub and Pecsok mention certain imperfections of I-131 as a tracer, and emphasize the necessity of using a carrier in many types of tests. The writers agree as to the importance of preliminary tests to ascertain "take-up" by various chemical, biochemical, and physical processes. Only by such investigation is it possible to ascertain the proper amount of radio tracer and carrier for a particular test.

Although I-131 undoubtedly is not the hypothetical "perfect tracer," under some conditions it has properties that are not far from ideal. In tests in the 6,400-ft pipe previously mentioned, a 90% recovery of I-131 was obtained in a run in which no carrier was added. With 10 ppm of NaI the recovery was virtually 100%. In runs with P-32, on the other hand, recoveries of less than 25% were obtained even in runs involving high isotopic dilution. This undesirable characteristic would appear to limit the utility of phosphorus as a tracer, whether measured by radioactivity or by chemical tests mentioned by Mr. Ingersoll.

Corrections for "Transactions."—Within the parenthesis, line 1 following Eq. 4, should read "p is the ordinate of Fig. 2(a) * * *"; in the line following Eq. 4, change the first "$\bar{u}$" to "u"; in line 3, page 5, change the second "Eq. 4" to "Eq. 3"; in line 3, page 8, change "2-in." to "1.25-in."; in Table 2, Col. 2, change the decimal points to commas and change "218" to "2,180"; in Tables 2 and 3, delete the last column headed "Base length"; in Table 2, Col. 7, change the heading to read "Relative height of mode"; in the captions of Tables 3(a) and 3(b) change the two decimal points to commas; and in line 1, p. 12, change "Eq. 7" to "Eq. 8."